

Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

How do angles, the unit circle, and right triangles let us define and compute the six trigonometric functions for any angle?

Unit Overview

Unit 5 introduced trigonometry through angles, the unit circle, and right triangles. Angles can be measured in degrees or radians, and coterminal angles share the same terminal side. On the unit circle, $\cos \theta$ and $\sin \theta$ are simply the x- and y-coordinates of a point, which extends sine and cosine to any angle. Tangent and the reciprocal functions (cotangent, secant, cosecant) build on sine and cosine. For right triangles, SOH-CAH-TOA connects angles to side ratios.

$$\begin{aligned}\text{radians} &= \text{degrees} \cdot (\pi/180) \\ s &= r\theta \text{ } (\theta \text{ in radians}) \\ \sin \theta &= y, \cos \theta = x \text{ (unit circle)}\end{aligned}$$

Recip.	$\tan \theta = \sin \theta / \cos \theta$; $\cot \theta = \cos \theta / \sin \theta$; $\sec \theta = 1 / \cos \theta$; $\csc \theta = 1 / \sin \theta$
Coterm.	$\theta \pm 360^\circ n$ (or $\theta \pm 2\pi n$) — angles sharing the same terminal side
Triangle	SOH-CAH-TOA: $\sin = \text{opp/hyp}$; $\cos = \text{adj/hyp}$; $\tan = \text{opp/adj}$

Quick Reference

- ASTC rule: All positive in QI, Sine positive in QII, Tangent positive in QIII, Cosine positive in QIV.
- Reference angle: the acute angle between the terminal side and the x-axis — gives the magnitude of a trig value.
- Memorize exact values at 0° , 30° , 45° , 60° , 90° — every other angle's exact value comes from a reference angle.

Worked Examples**EXAMPLE 1**

Convert 150° to radians, and convert $7\pi/6$ radians to degrees.

- $150^\circ \cdot (\pi/180) = 5\pi/6$
- $(7\pi/6) \cdot (180/\pi) = 7 \cdot 30^\circ = 210^\circ$

Answer: $150^\circ = 5\pi/6$ radians; $7\pi/6$ radians = 210°

EXAMPLE 2

Find a positive and a negative coterminal angle for 40° .

- Add 360° : $40^\circ + 360^\circ = 400^\circ$
- Subtract 360° : $40^\circ - 360^\circ = -320^\circ$

Answer: 400° and -320° are coterminal with 40°

EXAMPLE 3

Find the arc length of a circle with radius 8 cm and central angle $2\pi/3$.

- Use $s = r\theta$ with θ already in radians
- $s = 8 \cdot (2\pi/3) = 16\pi/3$

Answer: $s = 16\pi/3 \approx 16.76$ cm

EXAMPLE 4

Find the exact values of $\sin(5\pi/4)$ and $\cos(5\pi/4)$.

- $5\pi/4$ is in Quadrant III; reference angle $= 5\pi/4 - \pi = \pi/4$
- In QIII, both sine and cosine are negative (ASTC)

Answer: $\sin(5\pi/4) = -\sqrt{2}/2$; $\cos(5\pi/4) = -\sqrt{2}/2$

EXAMPLE 5

In a right triangle, angle $A = 35^\circ$ and the hypotenuse = 20. Find the opposite and adjacent sides.

- SOH: opposite $= 20 \cdot \sin(35^\circ) \approx 20 \cdot 0.5736$
- CAH: adjacent $= 20 \cdot \cos(35^\circ) \approx 20 \cdot 0.8192$

Answer: opposite ≈ 11.47 ; adjacent ≈ 16.38

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

- 1** Convert 225° to radians, and convert $3\pi/4$ radians to degrees.

Hint: Multiply degrees by $\pi/180$, or multiply radians by $180/\pi$.

- 2** Find a positive and a negative coterminal angle for 100° .

Hint: Add 360° for a positive coterminal angle; subtract 360° for a negative one.

- 3** Find the arc length of a circle with radius 5 in and central angle $3\pi/4$.

Hint: Use $s=r\theta$ directly since θ is already in radians.

- 4** Find the exact values of $\sin(2\pi/3)$ and $\cos(2\pi/3)$.

Hint: $2\pi/3$ is in Quadrant II with reference angle $\pi/3$. Which functions are positive in QII?

- 5** A ladder makes a 60° angle with the ground and reaches 12 ft up a wall. Find the length of the ladder.

Hint: The 12 ft height is the side OPPOSITE the angle; the ladder is the hypotenuse. Use SOH.

Key Vocabulary

Radian	A unit of angle measure where one full revolution equals 2π radians (360°).
Coterminal Angles	Angles that share the same terminal side, differing by a multiple of 360° (or 2π).
Reference Angle	The acute angle between an angle's terminal side and the x-axis; gives the magnitude of its trig values.
Unit Circle	A circle of radius 1 centered at the origin, where $\cos \theta$ and $\sin \theta$ are the x- and y-coordinates.
Periodic Function	A function that repeats its values at regular intervals (sine and cosine repeat every 2π).
Reciprocal Trig Functions	Cosecant ($1/\sin$), secant ($1/\cos$), and cotangent ($1/\tan$).
Angle of Elevation/Depression	The angle up (elevation) or down (depression) from horizontal to a line of sight.
Arc Length	The distance along a circle's edge: $s = r\theta$, with θ measured in radians.

Check Your Understanding

Circle the letter of the correct answer for each question.

- 1** Convert 60° to radians.

Multiple Choice

A $\pi/3$

B $\pi/6$

C $2\pi/3$

D $\pi/2$

2 Convert $3\pi/2$ radians to degrees.

Multiple Choice

A 90° **B** 180° **C** 270° **D** 360° **3 Find $\sin(\pi/6)$.**

Multiple Choice

A $1/2$ **B** $\sqrt{3}/2$ **C** $\sqrt{2}/2$ **D** 1**4 Which function is the reciprocal of cosine?**

Multiple Choice

A Tangent**B** Cotangent**C** Secant**D** Cosecant**5 In a right triangle with a 30° angle and hypotenuse 10, find the side opposite the angle.**

Multiple Choice

A 5**B** 8.66**C** 10**D** 3

Common Mistakes to Avoid

✗ Using a degree angle directly in a formula that requires radians (like $s=r\theta$).

✓ Always check the units first — arc length and unit-circle formulas require θ in radians.

✗ Mixing up sine and cosine on the unit circle.

✓ The point on the unit circle is $(\cos \theta, \sin \theta)$ — x comes first, matching cosine.

✗ Using a reference angle's trig value without applying the correct sign for the quadrant.

✓ Find the reference angle for magnitude, then use ASTC to determine the correct sign.

✗ Confusing angle of elevation with angle of depression, or mislabeling triangle sides.

✓ Elevation is measured UP from horizontal; depression is measured DOWN. Always draw a diagram first.

Math Tips

- ★ Degrees to radians: multiply by $\pi/180$. Radians to degrees: multiply by $180/\pi$.
- ★ Coterminal angles differ by a full revolution: add or subtract 360° (or 2π radians).
- ★ Memorize the unit circle values at 0° , 30° , 45° , 60° , 90° — everything else comes from a reference angle.
- ★ ASTC: All positive (QI), Sine positive (QII), Tangent positive (QIII), Cosine positive (QIV).
- ★ SOH-CAH-TOA: Sine=Opposite/Hypotenuse, Cosine=Adjacent/Hypotenuse, Tangent=Opposite/Adjacent.